I547: Audio Music Processing
Homework 4

1. Consider the first (y) and second (z) "differences" of a sequence x:

$$\begin{aligned} y_j &= x_j - x_{j-1} \\ z_j &= y_j - y_{j-1} \end{aligned}$$

   for $j = 1, 2, \ldots$. These are the discrete analogs of the first and second derivatives.

   (a) Write $y = a * x$ and $z = b * x$ where a and b are short filters.

   (b) Compute the frequency responses of these filters by padding them with 0's and taking the FFT. Which frequencies of the signal x are suppressed and amplified by using these filters? Justify your response using plots of the frequency response of the filters.

2. We know that the FFT transforms convolution into "pointwise" multiplication. The reverse relationship:

$$x \cdot a \overset{\text{FFT}}{\leftrightarrow} X * A$$

   also holds where $x \overset{\text{FFT}}{\leftrightarrow} X$ and $a \overset{\text{FFT}}{\leftrightarrow} A$ and $\cdot$ denotes pointwise multiplication.

   (a) What is the FFT of an N-point Hann (raised cosine) window for *all* values of the frequency variable. You don't need to calculate to obtain this answer.

   (b) Suppose we have a sine wave that oscillates an integral number of times per N points. What is the effect in the frequency domain of multiplying the sine by the Hann window function before taking the FFT (i.e. the usual windowing procedure)?

3. Initialize y_1 and y_2 and define y_j for $j = 3, 4, \ldots$ by

$$y_j = 1.5 y_{j-1} - .8 y_{j-2}$$

   The resulting y can be written as $y_j = a r^j \sin(j\theta + \phi)$. Solve for r and θ.

4. Write an R program to filter the bass oboe data using a filter that "passes" only frequencies between 1000 and 3000 Hz. Listen to the result.

5. Consider the random walk model

$$y_i = y_{i-1}/2 + \epsilon_i$$

   where $\epsilon = (\epsilon_1, \epsilon_2, \ldots)$ is a vector of independent 0-mean random numbers (eg rnorm(n)).

   (a) Write this equation as

$$a * y = \epsilon$$

   for some filter a where $*$ denotes convolution.

   (b) Express the FFT of y in terms of the transforms of a and ϵ

   (c) Describe the spectral qualities of y. That is, compare the energy present at different frequencies.

   (d) Create y by setting the first value to 0 and running the eqaution. Listen to the result, and compute the FFT.

6. Imagine a simple room that produces an echo. The echo comes $D = .125$ secs after the original sound, but decreases in amplitude by a factor of $F = .8$. This echo is now *part* of the sound in the room, so D seconds later we will hear another version of the original echo, decreased in amplitude by another factor of F. This process will continue indefinitely, though the echos eventually become inaudible as their amplitudes decrease. Explicitly model this process by an AR filter with input signal x and output signal y, for audio data sampled at 8kHz. Run this on the bass oboe data to see the effect.