I547: Audio Music Processing
Homework 1 solutions

1. Using R, create a sine tone at 440 Hz (tuning a) sampling at 8KHz. Make sure that your result is right by using a tuning fork, pitch pipe or some other standard. (The telephone dial tone is A440 and F below). Compare this with both cosine wave also at 440 Hz and the sine wave at -440 Hz. Describe the similarities and differences between the sounds and explain their origin.

```
> sr = 8000
> f = 440
> t = seq(0:3,1/sr)
> y = sin(2*pi*f*t)  # or cos, or -f
```

   The results sound identical (except for possible clicking at start of file) since sin, cos, and -sin are related by a phase shift.

2. The $*$ operation in R is pointwise multiplication between vectors (of the same length). Thus (1:10) * (1:10) gives a vector of the 1st 10 squares. Use this idea to create a sine tone with "amplitude vibrato" — a periodic cycling of the (maximum) amplitude of the wave. Make the vibrato at 5 Hz.

```
> sr = 8000
> f = 440
> v = 5
> t = seq(0:3,1/sr)
> a = 1 + .5*sin(2*pi*v*t)
> y = a * sin(2*pi*f*t)
```

3. Consider the R program for intervals discussed in class with the sampling rate changed to 8KHz. Using the interval of an octave (c = 2), generate 8 notes starting at 440 Hz. What are the frequencies of the notes you hear? Explain how you arrived at your answer.

   440, 880, 1760, 3520, 960, 1920, 3840, 220, 440, 880

4. (a) Using the **repeated_intervals.r** program, create a sequence of 12 notes that divide the octave into 12 equal musical intervals. The result is called an equal tempered chromatic scale, as discussed in class.

   (b) Alternatively, create a different 12-note chromatic scale in the following way: Begin with the same starting frequency, f_0, you used in your chromatic scale above. Generate 12 more pitches by moving up by a "perfect fifth" (a 3/2 ratio) between successive pitches, bringing each pitch back into the range $(f_0, 2f_0)$ by moving down by octaves, as necessary. The final pitch you generate should be about an octave above the initial pitch. Order the resulting 13 pitches in increasing order, and compare the two chromatic scales by listening. Comment on any differences you can hear.

   To divide the octave into 12 equal intervals we must take the frequencies:

$$f, f \times 2^{1/12}, f \times 2^{2/12}, \ldots, f \times 2^{12/12} = 2f$$

   Thus in the program we take

```
c = 2^(1/12)
```

   For the second part of the problem the frequencies can be created by:

```
p = rep(0,12);
f = 440;
g = f;
for (i in 1:12) {
    p[i] = g;
   g = 3*g/2;
   if (g > 2*f) g = g/2
}
p = sort(p);
```

5. Suppose we have the collection of rational numbers $S = \{0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}\}$ and a period length $p > 0$. We generate a periodic rhythm by tapping at the times $\{p(n+s) : n \in Z^+, s \in S\}$ where $Z^+ = \{0, 1, 2, 3, ...\}$. When p is on the order of 1 second we hear a rhythm. Describe precisely what you would hear if $p = .005$ using standard terminology from this course.

   The rhythm can be seen as a sum of two simple period patterns. The first divides the interval of length p into 3 equal pieces, while the second divides the interval into 2 equal pieces. Thus we hear two pitches, one at $2/.005$ Hz and one at $3/.005$ Hz. The result would be a perfect fifth.

6. Suppose you are given a starting frequency, f, and two operations that can be applied in any sequence desired.

   (a) You can move up a perfect fifth

   (b) You can move down an octave

   Show that it is not possible to generate the frequency that is exactly an octave above your starting point.

   If we were to create an octave above the starting frequency this would mean

$$2 = \frac{3^k}{2^{n+k}}$$

   where $k, n \geq 0$. Clearly this is not possible

7. Create a pitch vibrato at 5 Hz by gradually varying the frequency of a sine tone.

   One way is ...

```
> sr = 8000
> f = 440
> v = 5
> t = seq(0:3,1/sr)
> y = sin(2*pi*f*t + 5*sin(2*pi*v*t))
```